Phase Diagram of Neutron-Proton Condensate in Asymmetric Nuclear Matter

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We investigate the phase structure of homogeneous and inhomogeneous neutron-proton condensate in isospin asymmetric nuclear matter. At extremely low nuclear density the condensed matter is in homogeneous phase at any temperature, while in general case it is in Larkin-Ovchinnikov-Fulde -Ferrell phase at low temperature. In comparison with the homogeneous superfluid, the inhomogeneous superfluid can survive at higher nuclear density and higher isospin asymmetry.

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It is well-known that the neutron-proton (np) pairing plays an important role in nuclear physics and astrophysics, such as the structure of medium-mass nuclei produced in radioactive nuclear beam facilities[1], the deuteron formation in intermediate energy heavy-ion collisions[2], the pion and kaon condensation[3], the r-process[4, 5], and the cooling of neutron stars. The microscopic calculations show that the nuclear matter supports np Cooper pairing in the $^3\mathrm{S}_1-^3\mathrm{D}_1$ channel due to the tensor component of the nuclear force, and the pairing gap is of the order of 10 MeV[6, 7, 8, 9, 10, 11, 12] at the saturation nuclear density. At low enough density the np Cooper pairs would go over to Bose-Einstein condensation(BEC) of deuterons in symmetric nuclear matter[2, 9].

The emergence of isospin asymmetry will generally suppress the np pairing, and the condensate will disappear when the asymmetry becomes sufficiently large. Near the saturation density, the np pairing correlation depends crucially on the mismatch between the two Fermi surfaces, and a small isospin asymmetry can break the condensate due to the Pauli blocking effect. At very low density, when neutrons and protons start to form deuterons and when the spatial separation between deuterons and between deuterons and neutrons is large, the Pauli blocking loses its efficiency in destroying a np condensate. In such situation, the isospin asymmetry can be very large, and the np condensate survives in the form of deuteron-neutron mixture in momentum space[13, 14]. Different from the symmetric nuclear matter where the thermal motion destroys the np condensate, for asymmetric nuclear matter the temperature effect will melt the condensate on one hand and increase the overlapping between the two effective Fermi surfaces on the other hand. As a result of the competition, in a wide density regime the temperature dependence of the superfluidity is very strange[15, 16]: The maximum condensate is not located at zero temperature, and the pairing even occurs only at intermediate temperature for large isospin asymmetry.

The above results are obtained by assuming the condensate is homogeneous in the ground state. What is the true phase structure of np condensate with isospin asymmetry when the inhomogeneous Larkin-Ovchinnikov-Fulde -Ferrell(LOFF) phase[18] is taken into account? How will the LOFF phase change the strange temperature behavior of np condensate found in [15]? In fact, there should exist a rich phase structure in asymmetric nuclear matter, since the isospin asymmetry essentially plays the same role as the population imbalance in the two-component resonantly interacting atomic Fermi gas[17]. Different to the cold atoms , in nuclear matter the phase separation at large length scale may be forbidden and the LOFF phase may be energetically favored. Although the LOFF phase was discussed in asymmetric nuclear matter at saturation density[19], the phase structure including the LOFF phase in the whole density, temperature and isospin asymmetry space and the effect of the LOFF phase on the strange temperature behavior of the np condensate are still unknown. We will present in this paper the phase diagrams in the density, temperature and isospin asymmetry space.

The often used formulae for superfluid in nuclear matter are discussed in detail in [20] where the superfluid state is described by a normal and an anomalous nucleon distribution functions $\mathcal{F}$ and $\mathcal{G}$. Generally, they are functions of momentum $\mathbf{p}$ and matrices in spin and isospin space. The formulae can be easily generalized to study the isospin asymmetric superfluid with total pair momentum $2\mathbf{q}$. We will start with the LOFF phase, and the homogeneous phase can be recovered by taking $\mathbf{q}=0$. Like the studies in [13, 14, 15, 16, 19], we discuss the np pairing in the $^3\mathrm{S}_1-^3\mathrm{D}_1$ channel with total spin $S=1$, isospin $T=0$ and their projections $S_z=T_z=0$. In this case the distribution functions take the structure

$$\begin{aligned}\mathcal{F}(\mathbf{p}) &= \mathcal{F}_{00}(\mathbf{p})\sigma_0\tau_0+\mathcal{F}_{03}(\mathbf{p})\sigma_0\tau_3,\\ \mathcal{G}(\mathbf{p}) &= \mathcal{G}_{30}(\mathbf{p})\sigma_3\sigma_2\tau_2,\end{aligned}\tag{1}$$

where σ_i and τ_i are the Pauli matrices in spin and isospin spaces. Using the minimum principle of the thermodynamic potential and the procedure of block diagonalization[21], we can express the elements as

$$\begin{aligned}\mathcal{F}_{00}(\mathbf{p}) &= 1/2-\xi_\mathbf{p}\left[1-f(E^+_\mathbf{p})-f(E^-_\mathbf{p})\right]/(2E_\mathbf{p}),\\ \mathcal{F}_{03}(\mathbf{p}) &= \left[f(E^-_\mathbf{p})-f(E^+_\mathbf{p})\right]/2,\\ \mathcal{G}_{30}(\mathbf{p}) &= -\Delta_\mathbf{p}\left[1-f(E^+_\mathbf{p})-f(E^-_\mathbf{p})\right]/(2E_\mathbf{p})\end{aligned}\tag{2}$$

with the notations $\xi_\mathbf{p}=\left(\mathbf{p}^2+\mathbf{q}^2\right)/(2m)-\mu$, $E_\mathbf{p}=$

$\sqrt{\xi_{\mathbf{p}}^2+\Delta_{\mathbf{p}}^2}$ and $E_{\mathbf{p}}^{\pm} = E_{\mathbf{p}} \pm (\delta\mu + \mathbf{p}\cdot\mathbf{q}/m)$, where m is the effective nucleon mass in the medium and $f(x) = 1/(e^{x/T}+1)$ is the Fermi-Dirac function with T being the temperature. We have introduced the average chemical potential $\mu = (\mu_n+\mu_p)/2$ and the mismatch $\delta\mu = (\mu_n-\mu_p)/2$ instead of the neutron and proton chemical potentials μ_n and μ_p. We have also neglected the possible neutron-proton mass splitting induced by the isospin asymmetry which is believed to be small. The np condensate $\Delta_{\mathbf{p}}$ is generally momentum dependent and satisfies the gap equation

$$\Delta_{\mathbf{p}} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} V(\mathbf{p},\mathbf{k})\mathcal{G}_{30}(\mathbf{k}), \tag{3}$$

where V is the nucleon-nucleon (NN) interaction potential. The LOFF momentum $\mathbf{q}$ should be determined via minimizing the the free energy $\mathcal{E}$, which ensures the total current $\mathbf{j}_s$ in the ground state to be zero,

$$\rho|\mathbf{q}| - 4\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{\mathbf{p}\cdot\mathbf{q}}{|\mathbf{q}|}\mathcal{F}_{03}(\mathbf{p}) = 0, \tag{4}$$

where we have used the total nucleon density $\rho = \rho_n+\rho_p$ and the isospin density asymmetry $\delta\rho = \rho_n - \rho_p$ instead of the neutron and proton densities ρ_n and ρ_p,

$$\rho = 4\int \frac{d^3\mathbf{p}}{(2\pi)^3}\mathcal{F}_{00}(\mathbf{p}),\quad \delta\rho = 4\int \frac{d^3\mathbf{p}}{(2\pi)^3}\mathcal{F}_{03}(\mathbf{p}). \tag{5}$$

Once the NN potential V is known, we can solve the coupled set of gap equations (3) and (4) together with the density equations (5) at given temperature T, baryon density ρ or equivalently the Fermi momentum $k_F = (1.5\pi^2\rho)^{1/3}$ and isospin asymmetry $\alpha = \delta\rho/\rho$, and obtain all possible phases, namely the normal phase $\Delta_{\mathbf{p}} = 0$, the homogeneous superfluid phase $\Delta_{\mathbf{p}} \neq 0,\ \mathbf{q} = 0$ and the LOFF phase $\Delta_{\mathbf{p}} \neq 0,\ \mathbf{q} \neq 0$. By comparing their free energies we can determine the true ground state.

The details of the phase diagram depend on the NN potential V we will chose in the numerical calculations, however, the qualitative topology structure of the phase diagram does not depend on that. To show this, we analyze the stability of the homogeneous superfluid phase against the formation of a nonzero Cooper pair momentum. For this purpose, we investigate the response of the free energy $\mathcal{E}$ to a small pair momentum $\mathbf{q}$ via the small $\mathbf{q}$ expansion, $\mathcal{E}(\mathbf{q}) = \mathcal{E}(\mathbf{0}) + \mathbf{j}_s\cdot\mathbf{q}/m + \rho_s\mathbf{q}^2/(2m) + \cdots$, where $\mathbf{j}_s = m\partial\mathcal{E}/\partial\mathbf{q}$ is the total current which is proportional to the left hand side of (4), and ρ_s is just the superfluid density defined by $\rho_s = m\partial^2\mathcal{E}/\partial\mathbf{q}^2$ of which the explicit form reads

$$\rho_s = \rho + \frac{2}{m}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{\mathbf{p}^2}{3}\left[f'(E_{\mathbf{p}}^+) + f'(E_{\mathbf{p}}^-)\right]\Big|_{\mathbf{q}=0} \tag{6}$$

with the definition $f'(x) = df(x)/dx$. The current $\mathbf{j}_s$ vanishes due to the gap equation for $\mathbf{q}$, and the sign of ρ_s controls the stability of the homogeneous superfluid, i.e., a negative ρ_s means the LOFF phase has lower free energy than the homogeneous superfluid phase.

The momentum dependence of the gap function $\Delta_{\mathbf{p}}$ is normally rather weak in a wide momentum region[13], and we can approximately treat it as a constant Δ for a qualitative analysis. At zero temperature, once the isospin asymmetry is turned on, there must exist a sharp breached region where the quasiparticle energy $E_{\mathbf{p}}^- < 0$ with the necessary condition $\delta\mu > \Delta$, and the momentum integration in (6) can be analytically carried out,

$$\rho_s = \rho\left[1 - \frac{p_+^3\Theta(\mu_+) + p_-^3\Theta(\mu_-)}{3\pi^2\rho}\frac{\delta\mu}{\sqrt{\delta\mu^2-\Delta^2}}\right], \tag{7}$$

where $p_\pm = \sqrt{2m\mu_\pm}$ are possible gapless nodes with $\mu_\pm = \mu \pm \sqrt{\delta\mu^2-\Delta^2}$. At high density the matter is in BCS regime where $\delta\mu, \Delta \ll \mu$, and the breached region is $p_- < |\mathbf{p}| < p_+$. Since $p_\pm$ are close to the Fermi momentum k_F, the superfluid density should be negative since $\rho_s \simeq \rho(1-\delta\mu/\sqrt{\delta\mu^2-\Delta^2})$. On the other hand, at low enough density the matter is in BEC regime, the chemical potential μ becomes negative which leads to $\mu_- < 0$ and a reduced breached region $0 < |\mathbf{p}| < p_+$. In this case p_+ is much smaller than k_F and the superfluid density becomes positive. Therefore, at zero temperature the superfluid is expected to evolve from an inhomogeneous phase to the homogeneous phase when the nuclear density decreases, which is a general phenomenon for BCS-BEC crossover with population imbalance[17].

At finite temperature, the breached region is smeared due to the thermal excitation, and $\delta\mu > \Delta$ is not necessary. At the critical temperature T_c there should be a second order phase transition from the superfluid to normal state, and for temperature $T \lesssim T_c$ the pairing gap behaves as $\Delta(T) \propto (1-T/T_c)^{1/2}$ which leads to the regular behavior of the superfluid density $\rho_s(T) \propto (1-T/T_c) > 0$. This means the temperature tends to stabilize the homogeneous phase. Combining with the behavior of the superfluid density at zero temperature, the homogeneous phase at sufficiently low density will keep stable at any temperature below T_c, while at high density there must exist a turning temperature T_s where ρ_s changes sign, and the superfluid should be in inhomogeneous phase at low temperature $T < T_s$ and in homogeneous phase at high temperature $T_s < T < T_c$. Since the homogeneous state is unstable at low temperature, combining with the fact that the critical isospin asymmetry for the LOFF phase is much larger than the homogeneous phase[19], the strange temperature behavior of the pairing gap found in [15] is probably unrealistic.

We now move to numerical calculations. The Paris NN potential is often used to describe the nuclear structure and nucleon superfluidity, and describes well the BCS-BEC crossover of np condensate[2, 13]. Since the qualitative topology structure of the phase diagram does not depend on specific models, for the sake of simplicity, we

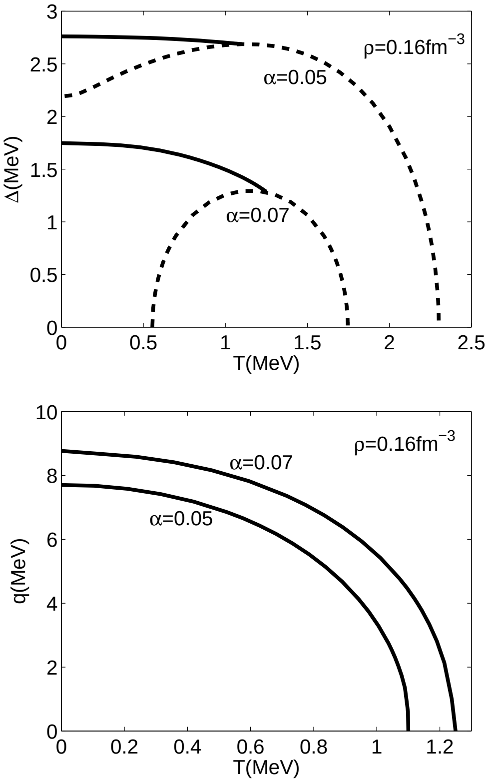

FIG. 1: The pairing gap Δ for homogeneous (dashed lines) and inhomogeneous (solid lines) condensates and LOFF momentum q as functions of temperature at normal density ρ_0 and for two values of isospin asymmetry.

employ a separable form of the Paris NN potential

$$V(\mathbf{r}_1, \mathbf{r}_2) = v_0 \left[1 - \eta \left(\frac{\rho\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right)}{\rho_0}\right)^{\gamma}\right] \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad (8)$$

which was developed in [22] to reproduce the pairing gap in $S = 1, T = 0$ channel and the bound state between zero energy and deuteron binding energy, where ρ_0 is the normal nuclear density and the parameters v_0, η, γ and an energy cutoff ε_c to regulate the model are determined by recovering the pairing gap in the realistic Paris NN potential. This separable form can also describe well the BCS-BEC crossover of np condensate[14]. In our numerical calculation, we choose $v_0 = -530$ MeV$\cdot$fm^3, $\eta = 0, \varepsilon_c = 60$ MeV and take m as the density-dependent nucleon mass corresponding to the Gogny force D1S[23]. We have checked that different parameter sets[22] lead to only a slight change in the numerical results.

In the low density BEC regime, the homogeneous phase is stable at any temperature below T_c, and the condensate is a regular decreasing function of temperature. Beyond this regime the strange phenomenon of the homogeneous condensate arises: The maximum pairing gap is located at non-zero temperature, and the condensate even appears only at intermediate temperature with two

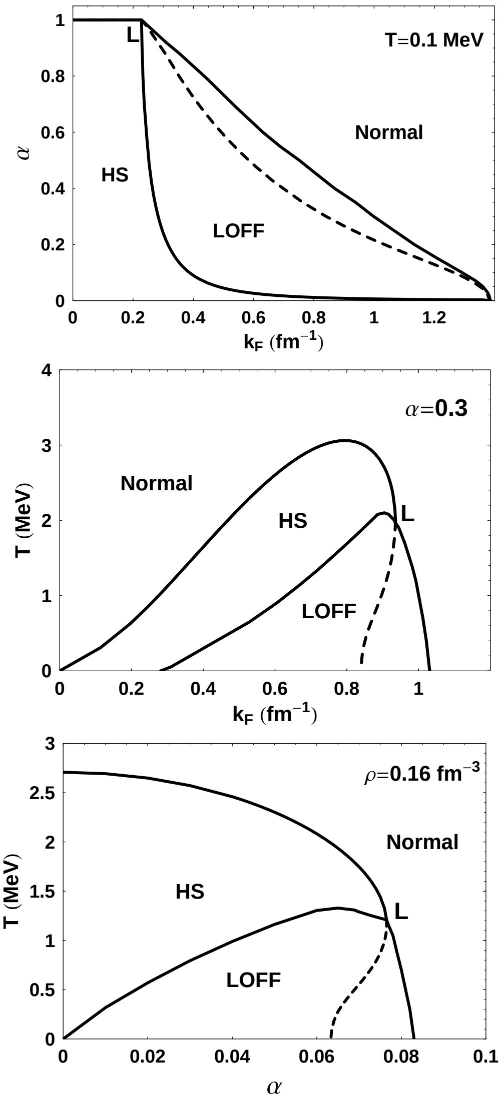

FIG. 2: The phase diagrams in $k_F - \alpha$, $k_F - T$ and $\alpha - T$ planes. In each plane, the labels HS, LOFF and Normal indicate homogeneous superfluid, LOFF superfluid and normal phase, the dashed line is the border of the unstable HS, and the three phases meet at a Lifshitz point L.

critical temperatures T_o and T_c for large isospin asymmetry, which is shown as dashed lines in the upper panel of Fig.1 for saturation density without loss of generality. However, the superfluid density ρ_s for the homogeneous phase is negative at $T < T_s$ and positive at $T_s < T < T_c$, where the turning temperature T_s is larger than the lower critical temperature T_o. This tells us that the homogeneous phase is stable at high temperature $T > T_s$ and unstable to formation of LOFF condensate at low temperature $T < T_s$. By calculating the LOFF pairing gap Δ and momentum $q = |\mathbf{q}|$ which are shown as solid lines in the upper and lower panels of Fig.1 and comparing the free energies for the homogeneous and LOFF states, the LOFF phase is energetically more favored than the homogeneous phase at $T < T_s$. Especially, different to

the homogeneous condensate, the LOFF condensate always starts at zero temperature. Therefore, after considering both homogeneous and inhomogeneous condensates, the strange temperature dependence of the pairing gap[15, 16] disappears and the condensate becomes a regular decreasing function of temperature. The LOFF momentum q, shown in the lower panel of Fig.1, drops down with increasing temperature and approaches to zero continuously at T_s, which indicates a continuous phase transition from homogeneous phase to LOFF phase. The continuity can be proven analytically. The gap equation (4) for q can be written as $qW(q) = 0$ with a trivial solution $q = 0$ for the homogeneous phase and a non-zero solution from $W(q) = 0$ for the LOFF phase. Using the expansion for $\mathcal{E}$ we find $\rho_s = W(0)$. Therefore, we must have $q = 0$ at $T = T_s$, providing that the LOFF solution is unique.

The phase diagrams of the np pairing are shown in Fig.2. We first discuss the one in the $k_F - \alpha$ plane at a very low temperature $T = 0.1$ MeV. When $\rho \to 0$ we find $\mu \to -\varepsilon_b/2$ at $\alpha = 0$ where ε_b is the deuteron binding energy, which means that the np condensate survives in the form of deuteron BEC. Consistent with the findings in atomic Fermi gas[17], the homogeneous phase (HS) is stable only at very low density BEC regime. In this regime, the critical isospin asymmetry can be very large, and even approaches to 1 for $k_F < 0.23$ fm^{-1}. Beyond this extremely low density regime, the superfluid density of the homogeneous phase becomes negative which indicates that the LOFF phase is energetically favored. By calculating the superfluid density of the HS phase and the LOFF solution, we can determine the phase boundaries between HS and LOFF and between LOFF and normal phase. The LOFF momentum is large at high ρ and high α and approaches to zero at the HS-LOFF boundary which means a continuous phase transition. If we consider HS only, the HS-Normal boundary (dashed line) is below the LOFF-Normal boundary, this shows that the introduction of LOFF phase enlarges the superfluid region. In the $k_F - T$ and $\alpha - T$ planes, the HS and LOFF phases are separated by the turning temperature T_s. Note that T_s starts at $k_F \neq 0$ in $k_F - T$ plane which corresponds to the stable HS at extremely low density but starts at $\alpha = 0$ in $\alpha - T$ plane for high density which means that the HS with small isospin asymmetry is easy to be stabilized. Again, in comparison with the calculation with only HS, the superfluid is extended to higher density or higher asymmetry region due to the introduction of the LOFF phase, and the unstable HS-Normal boundary (dashed lines) which reflects the strange "intermediate temperature superfluidity" is replaced by the LOFF-Normal boundary. The phase transitions in the three planes are all of second order, and in any case the three phases meet at a Lifshitz point L[24]. The $\alpha - T$ phase diagram we obtained is very similar to the generic phase diagram of two-component ultracold Fermi gas in a potential trap[25].

In summary, we have qualitatively investigated the phase structure of np condensate in isospin asymmetric nuclear matter and confirmed our analysis with a model NN potential. The important findings are: 1) The LOFF phase is the ground state in a wide region of nuclear density, temperature and isospin asymmetry, except for very low density and high temperature. 2) The strange temperature behavior of the np condensate is washed out by the LOFF phase at low temperatures. 3)The superfluid region is expanded to high density and high asymmetry due to the introduction of LOFF phase.

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